funRG with discrete symmetry-breaking

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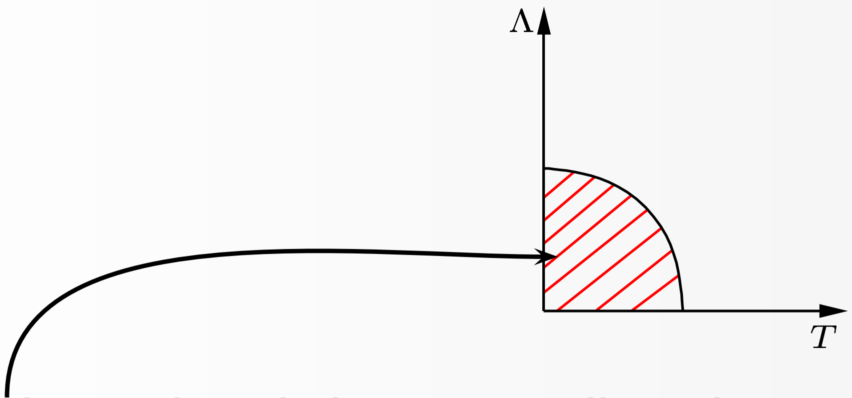
für Festkörperforschung

[1]now Würzburg [2]now Paris

Problem statement



- This region is inaccessible to symmetric-phase funRG techniques.
- Until recently, funRG techniques were unable to reproduce mean-field results for mean-field-exact models.
- BCS-model (U(1) symmetry) at $T = 0$ has been treated[1]

This talk: discrete-symmetry breaking, $T > 0$.

[1] Salmhofer, Honerkamp, Metzner, Lauscher 2004

Hamiltonian

- At half-filling: a repulsive interaction restricted to momentum-transfers of $Q := (\pi, \pi, \dots)$ generates a d-dimensional charge-density wave.

  $H = \sum_k \varepsilon_k n_k - \frac{U_0}{N} \sum_{k,k'} c_k^\dagger c_{k+Q} c_{k'}^\dagger c_{k'+Q} + \sum_k \Delta_{\text{ext}} c_{k+Q}^\dagger c_k$
- We assume the thermodynamic limit and a grand canonical ensemble at zero chemical potential (Here, this implies half filling).
- Δ/U_0: amplitude of the density wave. Appears as gap in the spectrum.
- U: effective interaction, effective coupling
- t: Hopping integral, unit of energy.

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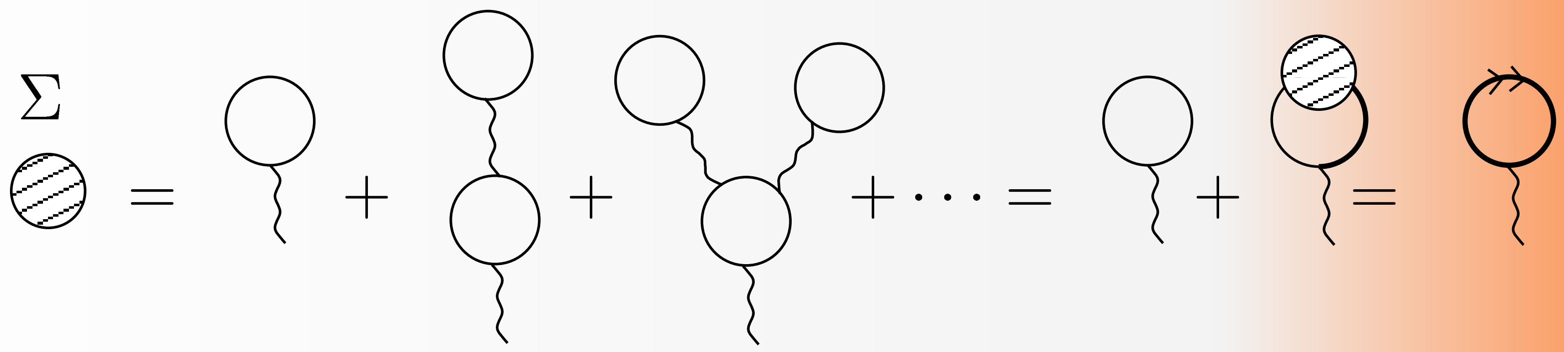
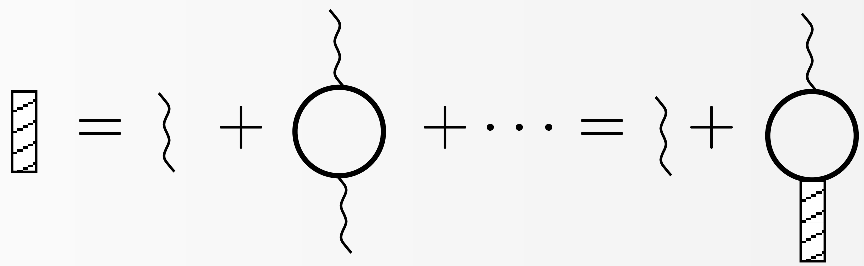
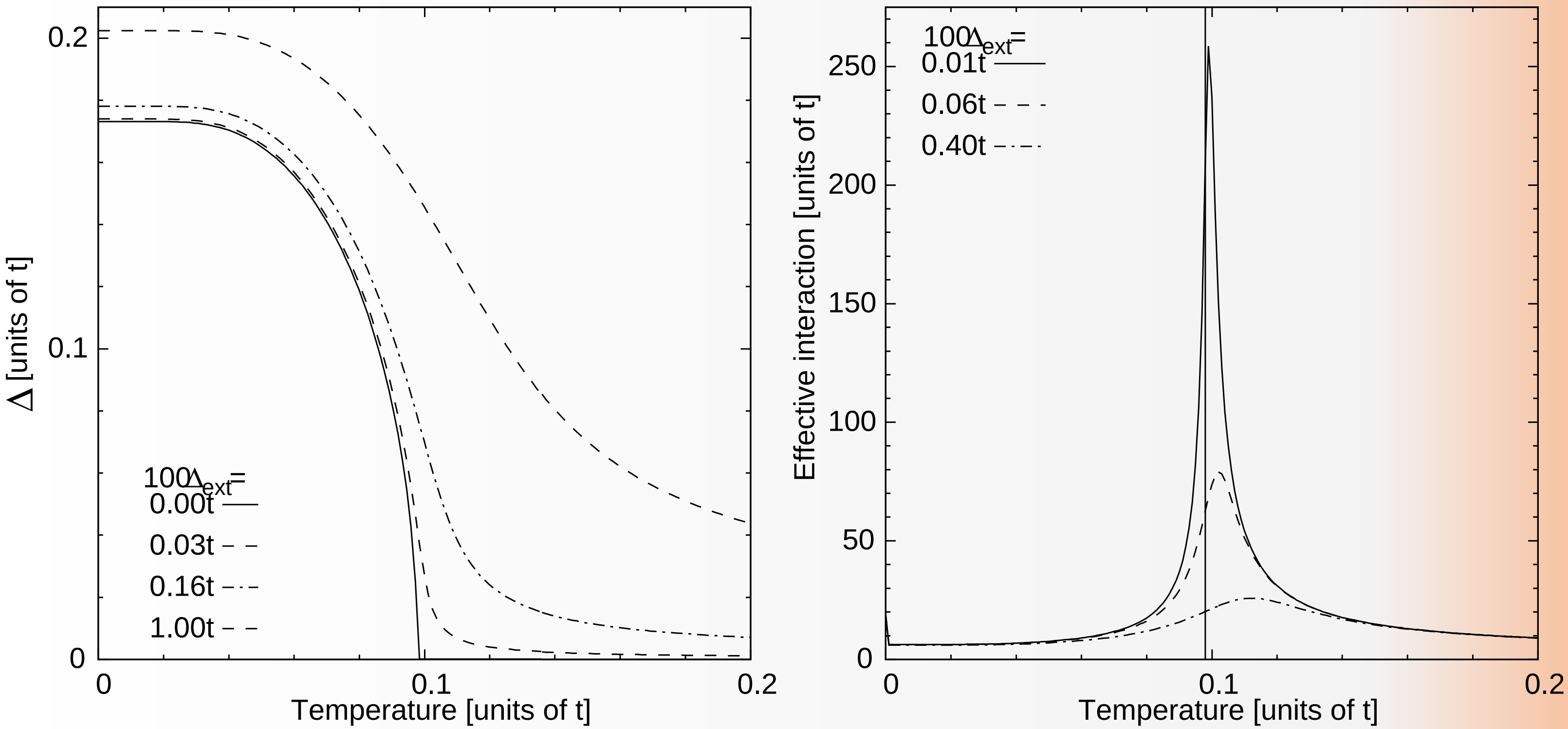
- Resumming perturbation theory leads to the gap equation.



- RPA resummation for the effective interaction.



Temperature-dependence of the gap, effective interaction

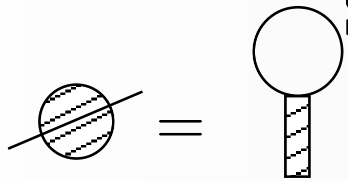


The phase transition is “smeared out” by the external field and the singularity of the effective interaction is regularized.

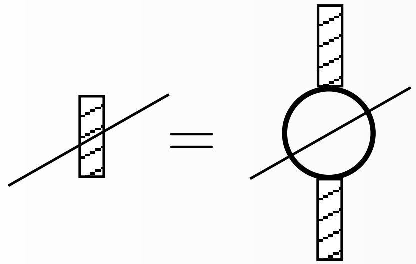
funRG equations

- Gap flow equation

$$\mathbb{S} = \mathbb{G}\frac{d}{d\Lambda}(\mathbb{G}_0^{-1})\mathbb{G}$$



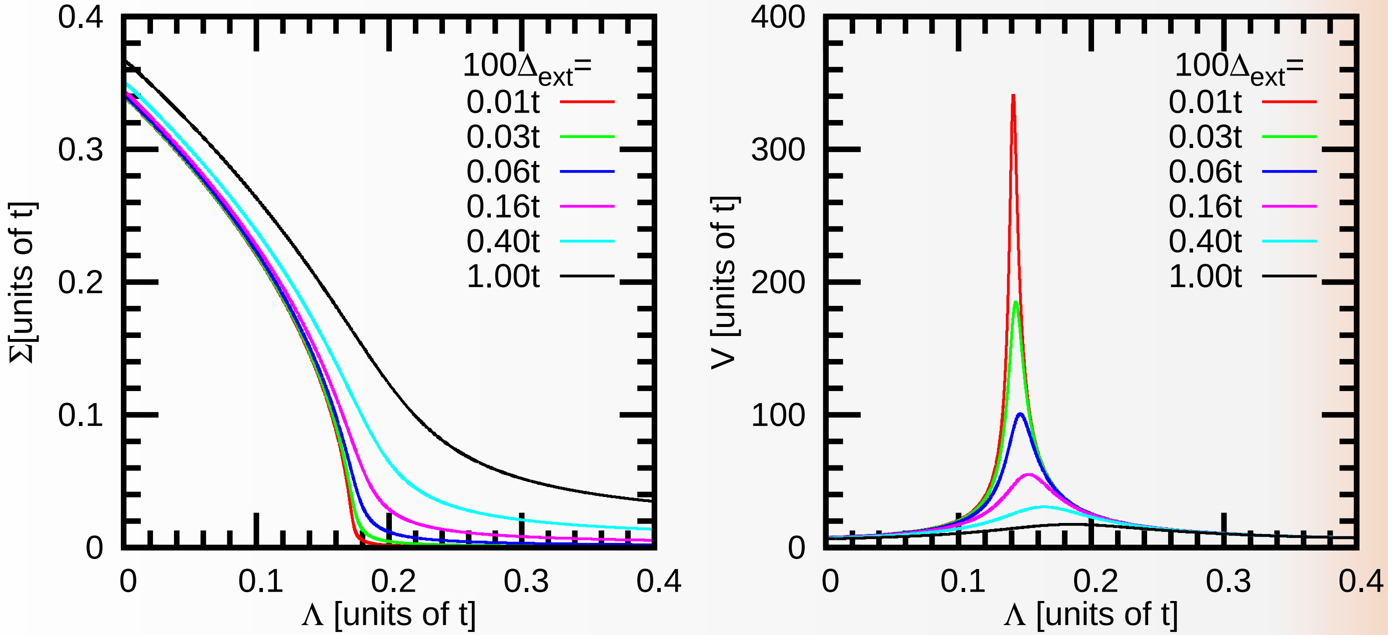
- Effective interaction flow equation



Initial conditions: $V_i = U_0$, $\Sigma_i = \Delta_{\text{ext}}$

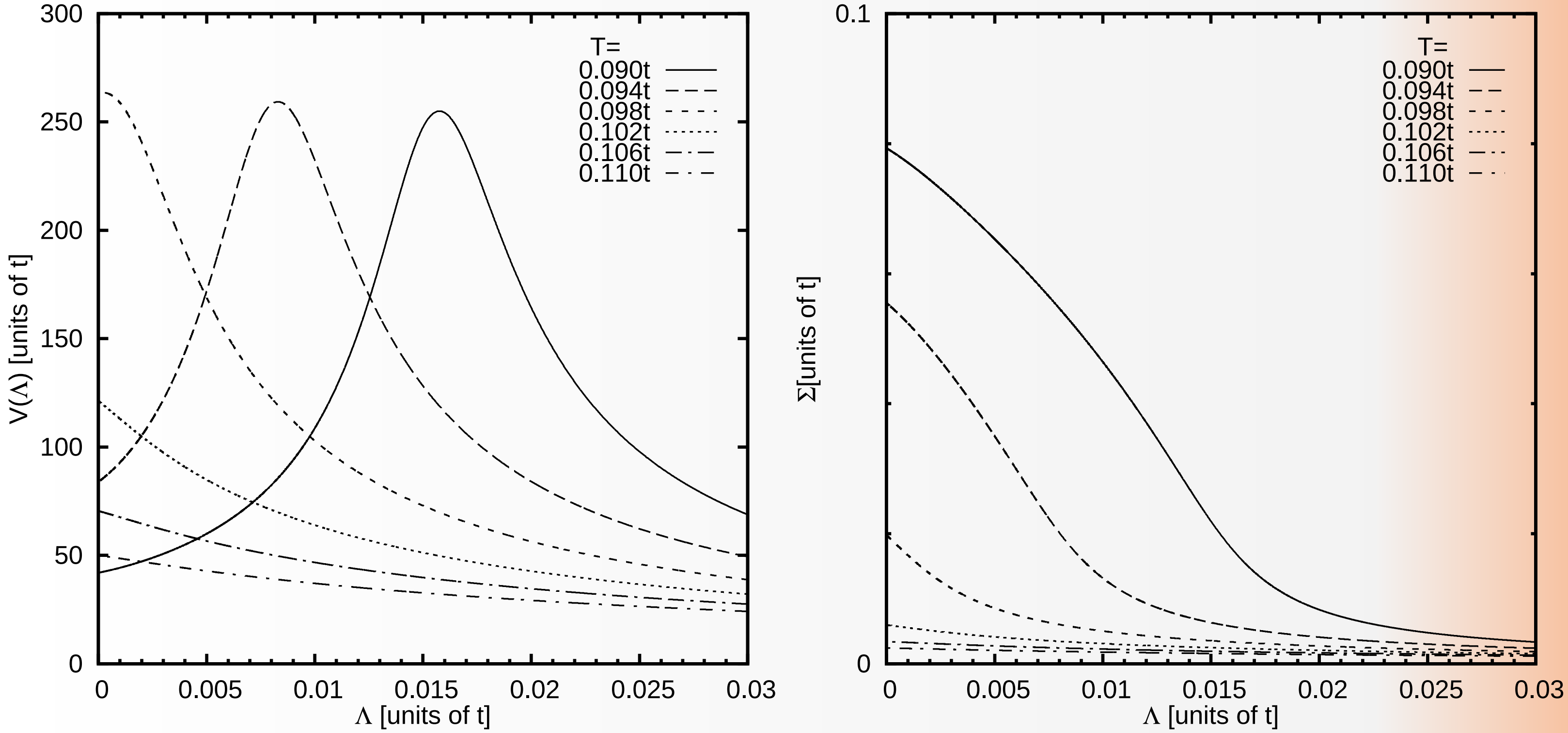
funRG flows at $T = 0$

- The sub-T_c flows of the effective interaction and the gap resemble the temperature dependences.



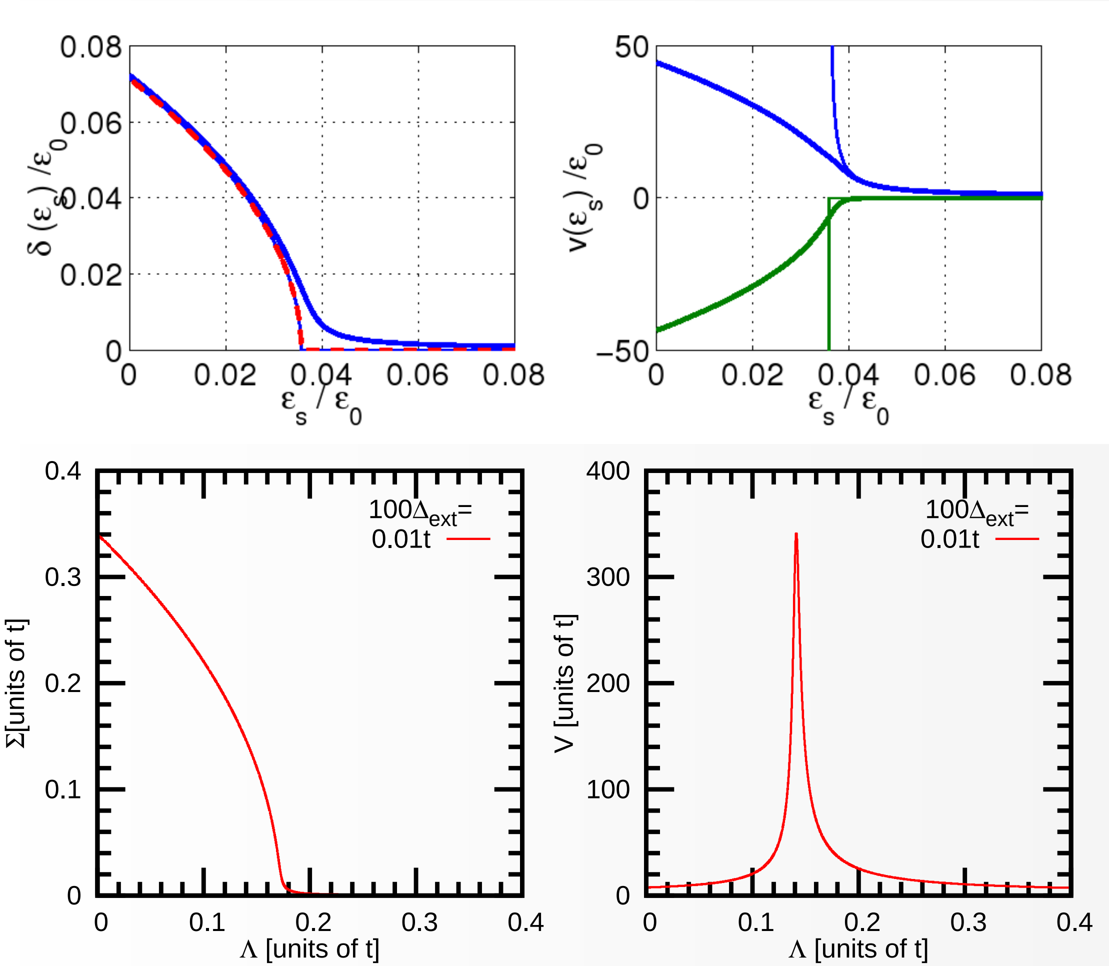
- Increasing the external field suppresses the effective interaction flow maximum and furthers the smearing of the transition.
- The self-energy's final value changes by only 10% while the initial gap varies over two orders of magnitude

funRG flows for $T > 0$



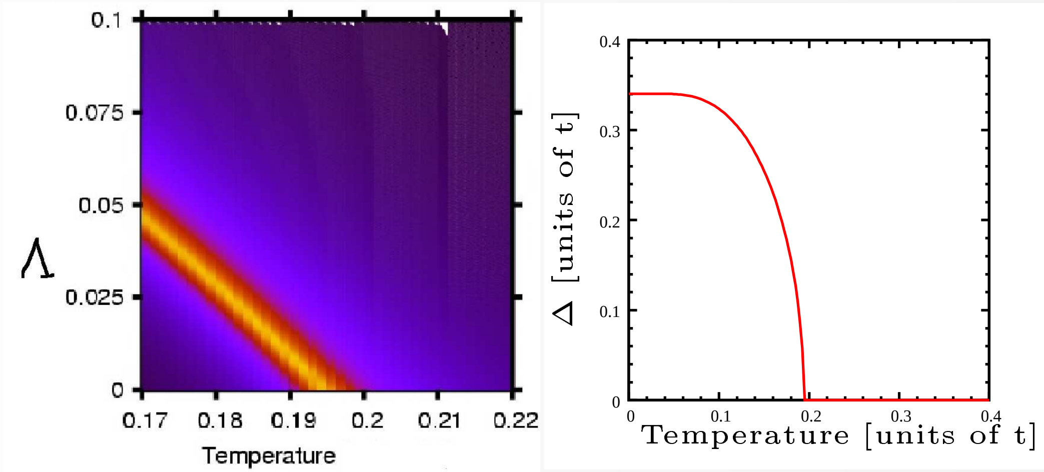
- Increase in temperature $\Rightarrow$ Graph of flow moves to lower scales

Comparison to BCS flows



BCS model: Normal and *anomalous* effective interactions (all arrows in/out) exist. Discrete-symmetry breaking flow of the effective interaction resembles the BCS flows' sum. Interpretation: amplitude mode.

T-dependence of Λ_c



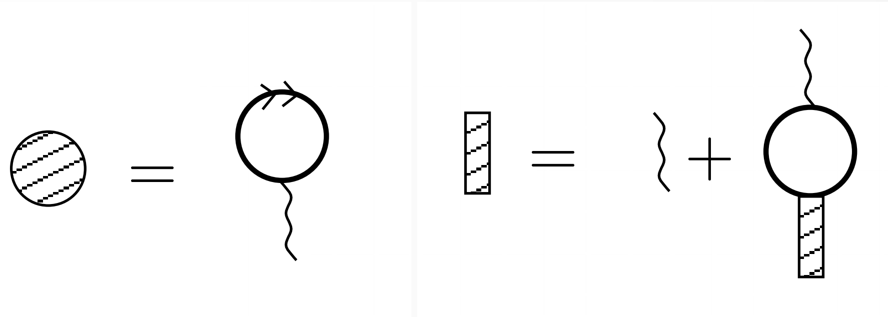
$$\Lambda_c \not\propto \Delta$$

Hamiltonian

$$H = \sum_k (\varepsilon_k - \mu) n_k - \frac{U_0}{N} \sum_{k,k'} c_k^\dagger c_{k+Q} c_{k'}^\dagger c_{k'+Q} + \sum_k \Delta_{\mathrm{ext}} c_{k+Q}^\dagger c_k$$

Exact Diagrammatics

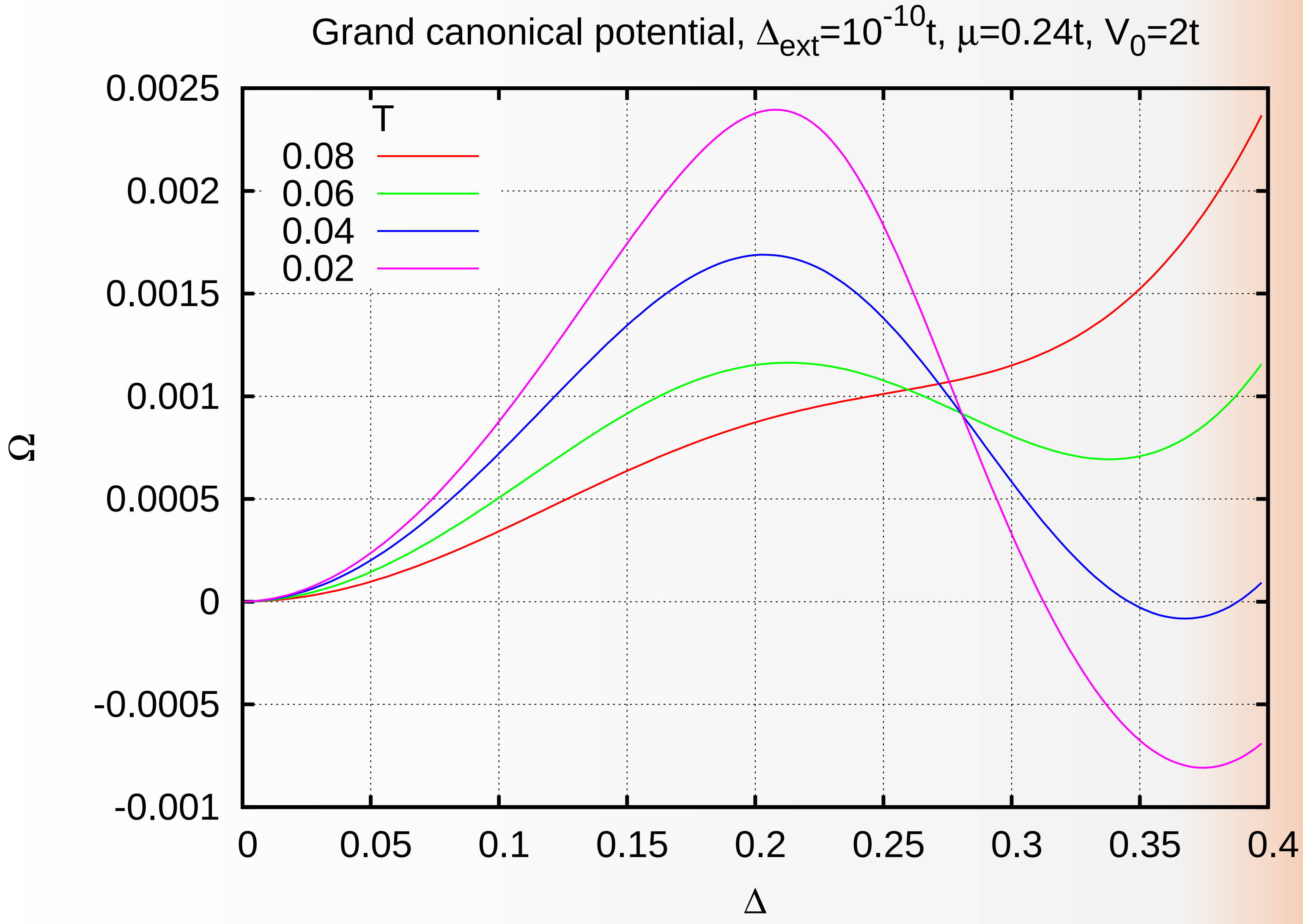
- Same diagrams as for $\mu = 0$



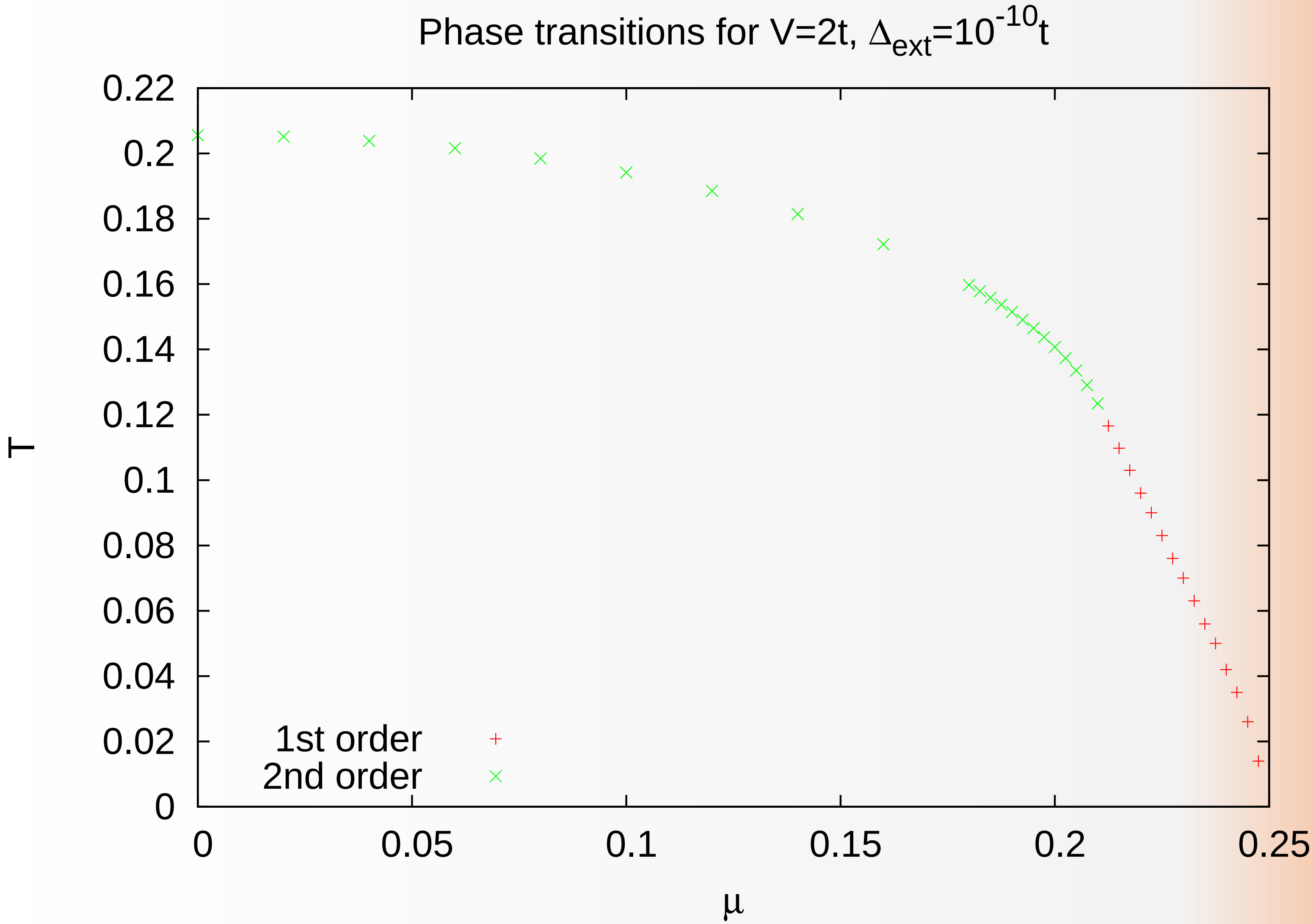
- Simple rule: Add $-\mu$ to the arguments of all Fermi distributions (and derivatives) in $\mu = 0$ equations.
- New: grand canonical potential Ω obtained by integrating the gap equation wrt the gap

$$\frac{\Omega}{\mathrm{Vol}} = \frac{\Delta^2}{2U_0} - \frac{1}{2}\sum_k \ln\left(f(-\mu - E)f(-\mu + E)\right)$$

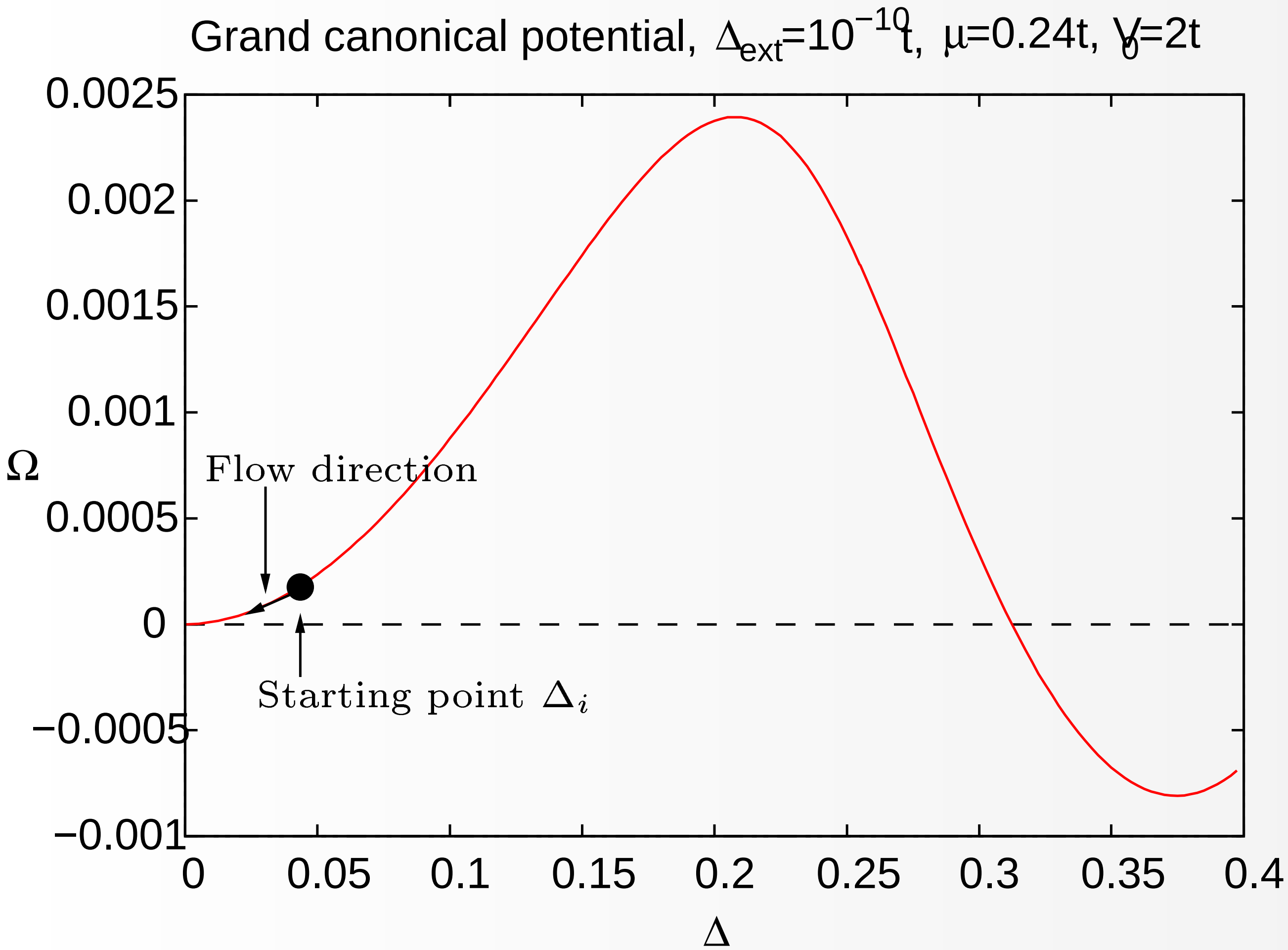
Grand canonical potential and phases



Phase diagram



Challenge



Challenge: Starting at large Δ *without* appreciably changing $\Omega(T)$.

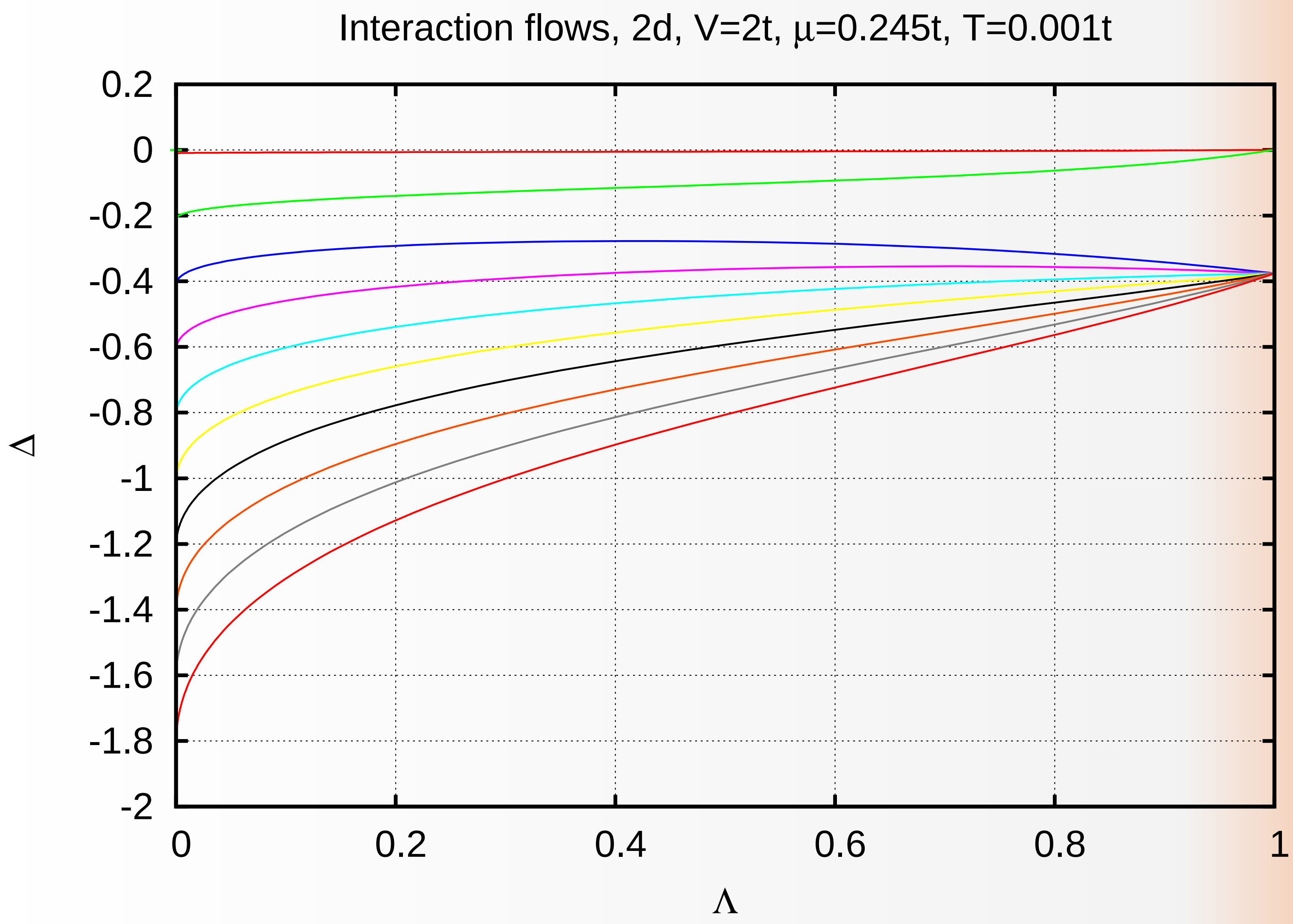
Counter terms and interaction flow

- Back to the Hamiltonian. Now, add a counterterm Δ_c.

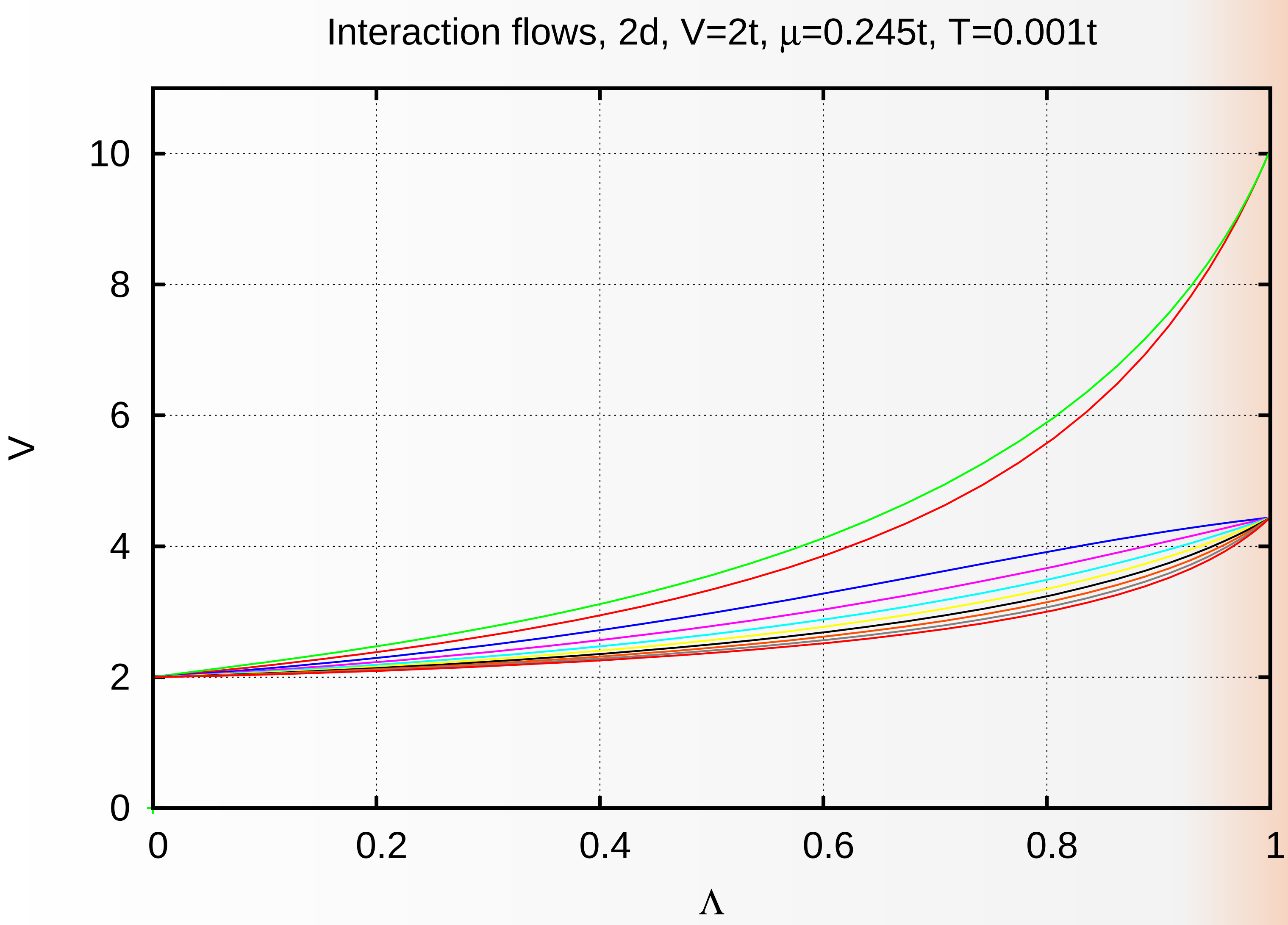
$$H = \sum_k (\varepsilon_k - \mu) n_k - \sum_k \Delta\Delta_{\mathrm{c}} c^\dagger_{k+Q} c_k$$
$$- \frac{U_0}{N} \sum_{k,k'} c^\dagger_k c_{k+Q} c^\dagger_{k'} c_{k'+Q} + \sum_k \Delta\Delta_{\mathrm{ext}} c^\dagger_{k+Q} c_k$$

- To naked propagator
- To initial self-energy
- Set $\Delta_{\mathrm{c}} = \Delta_{\mathrm{ext}}$
- Momentum-shell cutoff: Δ_{ext} and Δ_{c} cancel at all scales.
- Interaction "cutoff": Δ_{ext} and Δ_{c} cancel *only* at the end of the flow. This implies: The initial self-energy can be chosen arbitrarily without changing the physics!

Interaction flow of the self-energy



Interaction flow of the coupling



Grand canonical potential?

- $\tilde{\Omega} = \Omega/\mathrm{Vol}$, flow equation (traditional 1PI):
  $\dot{\tilde{\Omega}} = \frac{1}{2}\mathrm{Tr}\left(\frac{d}{d\Lambda}(G_0^{-1})(G - G_0)\right)$
- Mahan (slightly generalized): $\dot{\tilde{\Omega}} = 2\mathrm{Tr}(G\Sigma\dot{\chi}/\chi)$, same as above except for a constant factor.
- Numerics currently fail to reproduce exact grand canonical potential.

Conclusions

- Flows in phases with a broken discrete symmetry possible and illustrated above and below T_c. Exact results obtainable for mean-field models.
- 1st-order phase transitions treatable with funRG.
  1. So far only using the interaction flow
  2. Work in progress
- Outlook: Use the new methods study physically more relevant problems.

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