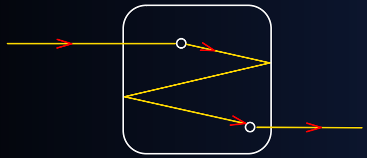
Point contacts in flat Dirac systems

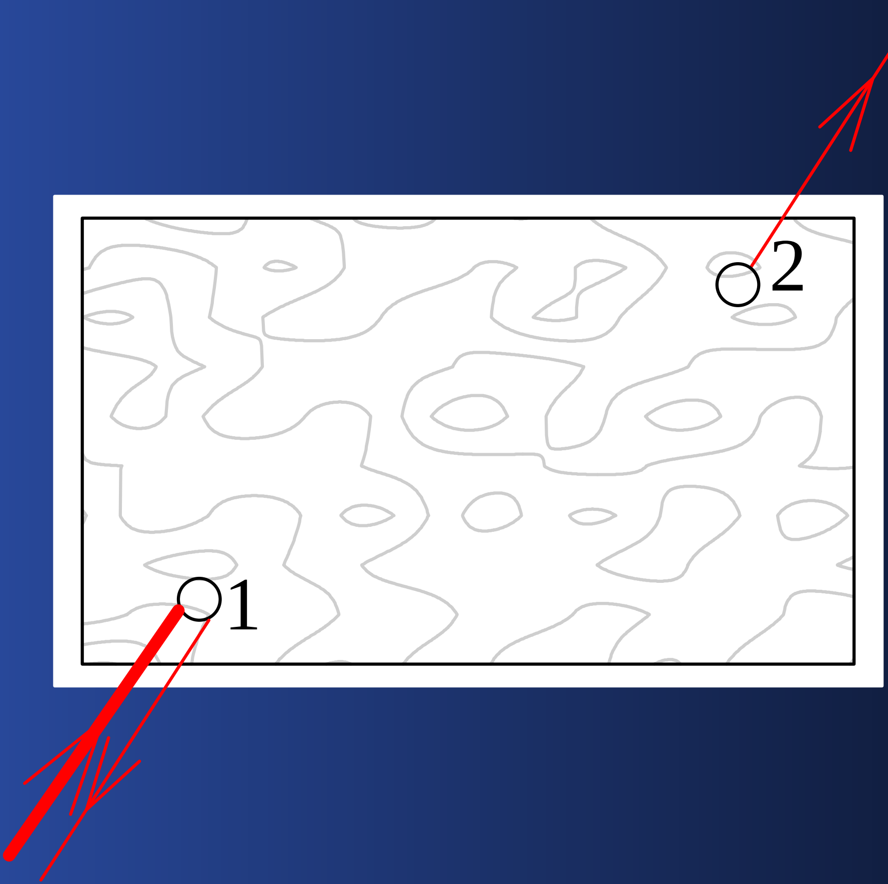
Roland Gersch

Institut für theoretische Physik der Universität zu Köln



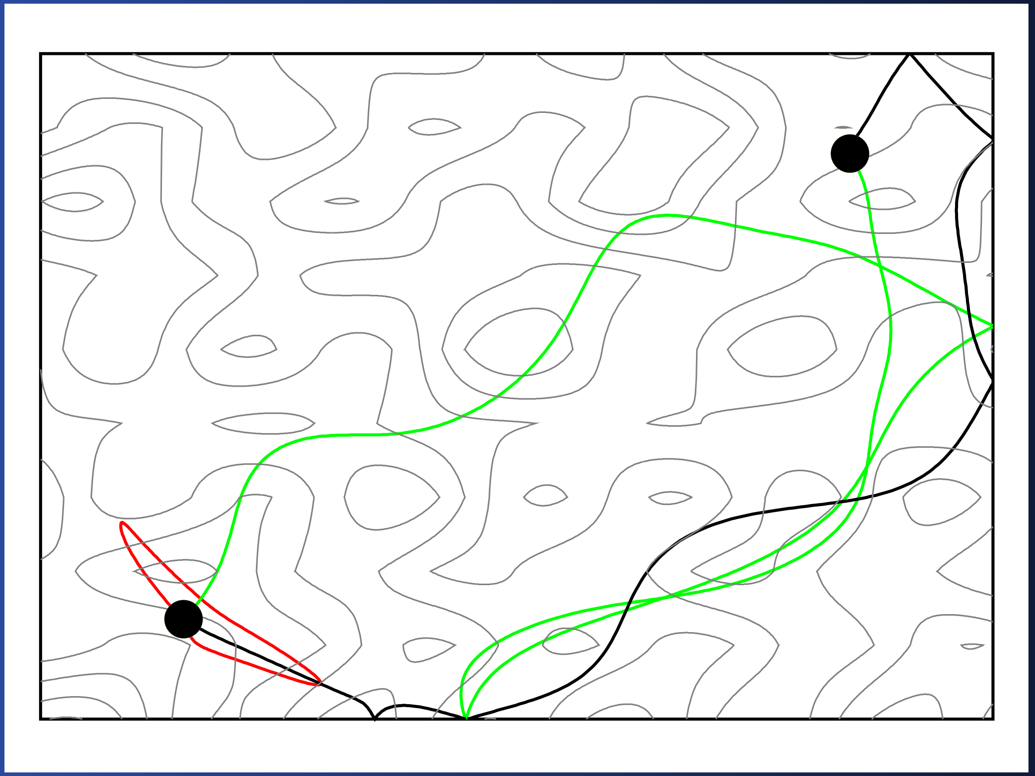
Definition

Consider a two-dimensional box: with random scalar potential, point contacts and attached leads.



Trajectories in the classical system

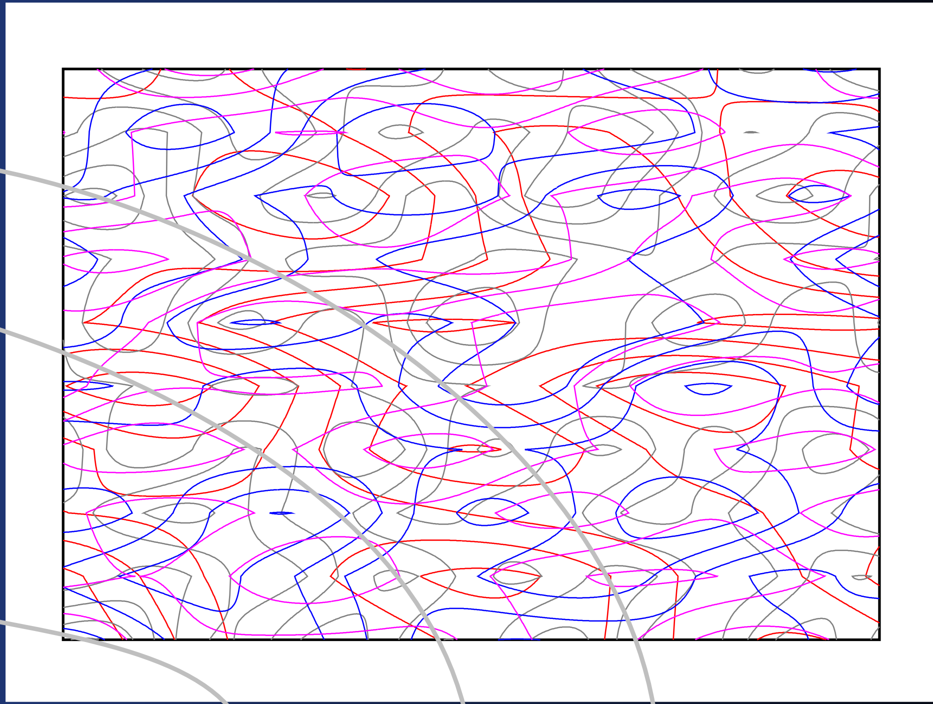
For given disorder, computers can calculate the trajectories:



Classically, point contacts and point contact conductances are intuitively defined.

Certain mesoscopic systems can be effectively described in the low temperature regime using Dirac operators with disorder in the

1. scalar potential V,
2. mass m,
3. magnetic gauge potential A_x, A_y.



$$D = (-i\partial_x + A_x)\sigma_x + (-i\partial_y + A_y)\sigma_y + m\sigma_z + V\mathbb{1}$$

Basic Quantum Mechanical Formalism

D fully characterizes the system. If

$$D\psi = E\psi,$$

then

$$\psi^\dagger \psi$$

is the probability density for finding a particle at the location given by the argument, while

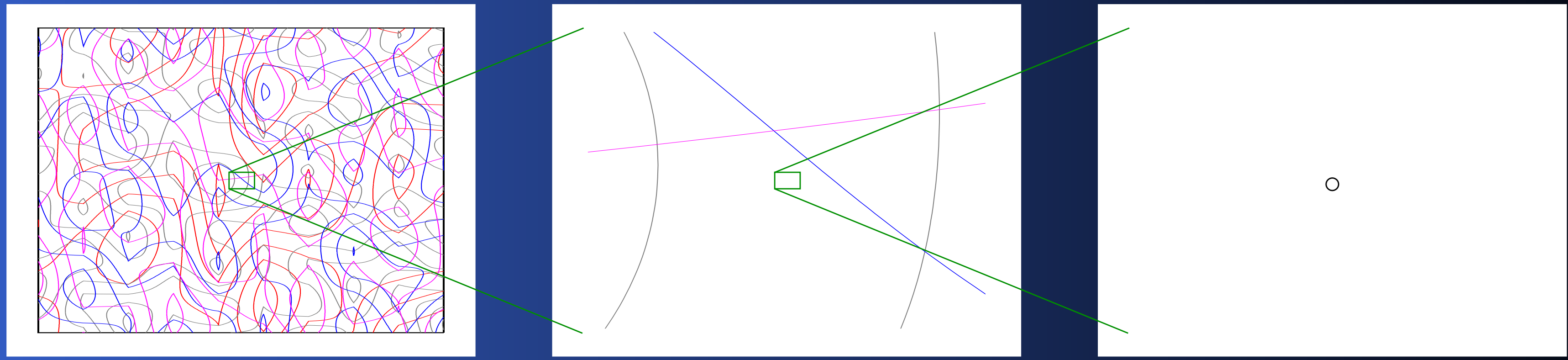
$$2\Re \psi^\dagger \vec{\sigma} \psi$$

is the vector field associated to the probability flux. Thus, the motion of a particle with energy E in the system can be predicted by solving an eigenvalue problem.

Our goal is to describe point contacts within this framework.

Ordering Disorder

Analysing an arbitrary disordered system is difficult.



$$\Rightarrow D = -i\partial_x\sigma_x - i\partial_y\sigma_y$$

Consider the plane minus a single point, o. The angular momentum $\mathcal{L}$ relative to o is conserved. Expanding a solution to last slide's eigenvalue problem into a series of $\mathcal{L}$-eigenspinors yields a separation of the solution space into two subspaces. One subspace has probability flowing into the system, the other has probability flowing out of the system.

Definition: Point Contact

There are two distinct types of point contacts:

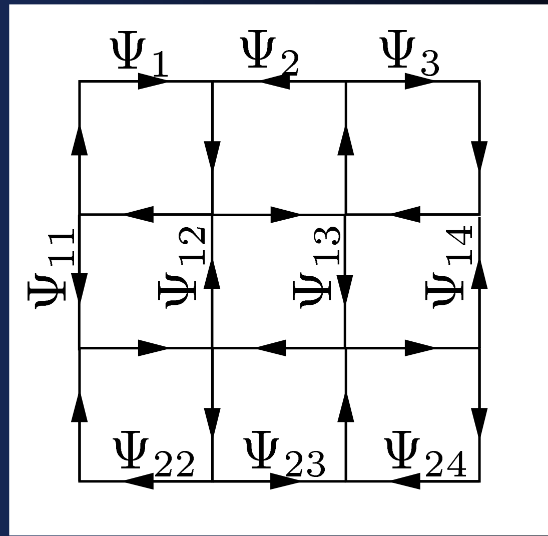
1. There is a net probability flux into (out of) the system.
2. There is no net flux, but the wave function still diverges at the contact due to a phase difference between incoming and outgoing part.

There is a point contact of type 1. or 2. if and only if

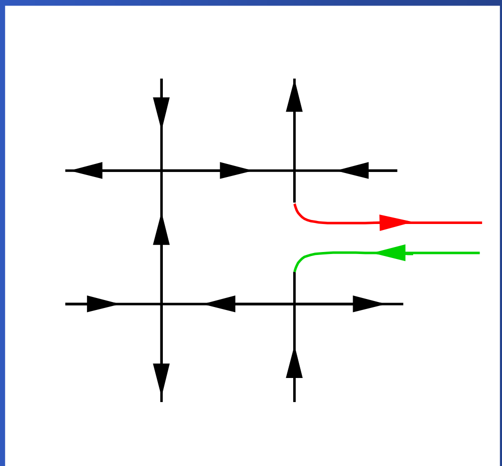
$$\lim_{r \to 0} \frac{\psi_1}{\ln(Er)} \neq 0$$

and finite.
Furthermore, the classically motivated angular momentum condition is also satisfied.

Point Contact in a Network Model

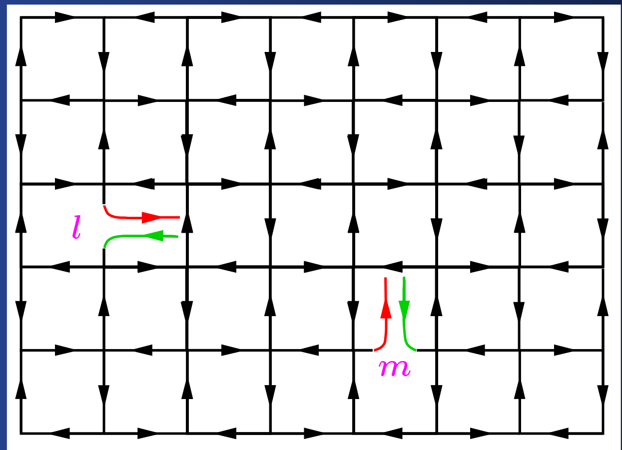


Consider a network model.



We add a point contact to the system by cutting a link open and attaching leads.

Conductance (Klesse and Zirnbauer)



Point contacts are added at links l and m. We are interested in the dependence of the disorder-averaged point contact conductance T_{lm} at energy E on the probability densities ρ_l, ρ_m of the network without contacts. We denote the disorder average by $\langle \dots \rangle$. Klesse and Zirnbauer find

$$2\pi\nu\langle \rho_m f(\rho_m/\rho_l)\rangle_e = \langle F(T_{lm})\rangle.$$

Conductance Formula in the Dirac Limit

In the vicinity of the network's critical point,

$$D = (-i\partial_x + A_x)\sigma_x + (-i\partial_y + A_y)\sigma_y + m\sigma_z + V\mathbb{1}$$

is an effective Hamiltonian if we take the continuum limit with respect to time and space and consider low energies. Instead of

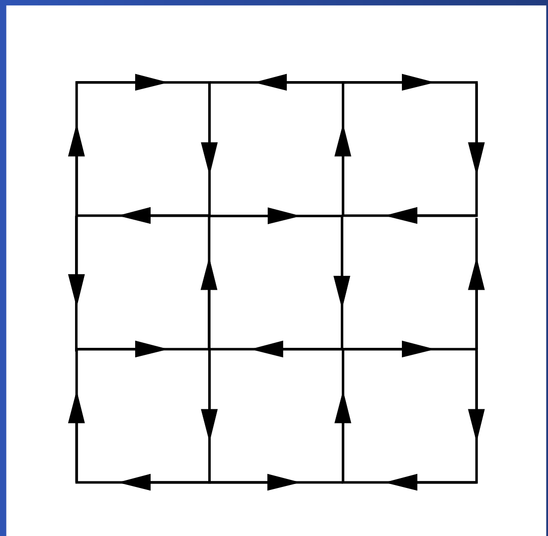
$$2\pi\nu\langle\rho_m f(\rho_m/\rho_l)\rangle_e = \langle F(T_{lm})\rangle$$

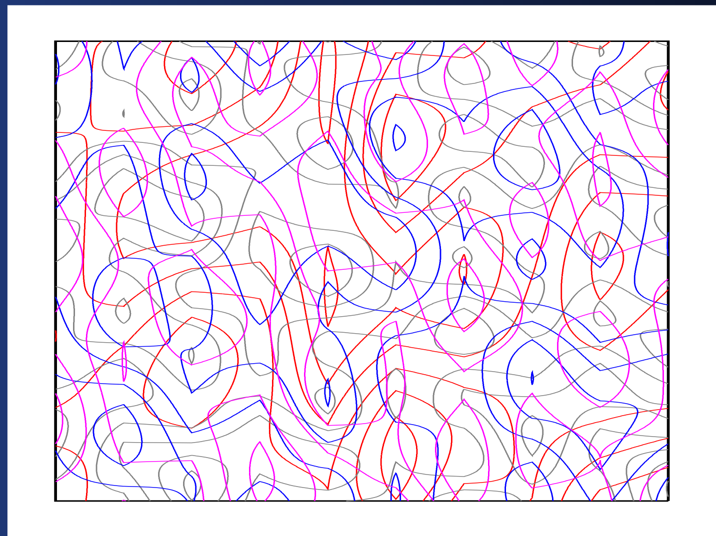
we find

$$\frac{1}{\langle\rho_m\rangle_e}\langle\rho_m f(\rho_m/\rho_l)\rangle_e = \langle F(T_{lm})\rangle.$$

Network Model versus Dirac Model

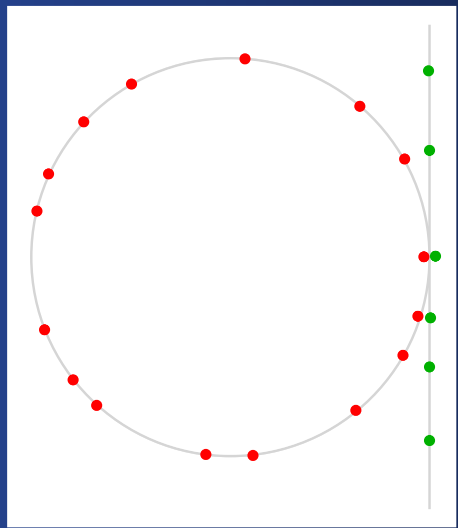
Network Model	v.	Dirac Model
arbitrary	v.	long wave disorder
Propagator	v.	Hamiltonian
periodic	v.	non-periodic spectrum





Network Model versus Dirac Model

Network Model	v.	Dirac Model
arbitrary	v.	long wave disorder
Propagator	v.	Hamiltonian
periodic	v.	non-periodic spectrum



Summary

We considered particles imprisoned in flat, two-dimensional boxes.

- In the classical case with random scalar potential, point contacts and conductances were intuitively defined.
- The Dirac system is described by an eigenvalue equation.
- Given sufficiently weak disorder, we can use boundary conditions to describe point contacts within this formalism.
- A conductance formula allows a comparison to a network model.

Thank you for listening!